A linear sequence has 4th and 5th terms like this

-9, -2, 5, 12, 19, 26, 33, 40 . . . . .

Find the first 8 terms of the sequence, the 100th term and the n^{th} term. $T_n = 7n - 16$

$T_{100} = 700 - 16$
$= 684$

One of the terms will be 61, Which one? $7n - 16 = 61$ $T_n = 7n - 16$

$n = 11$

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Sequences

1. Linear: The first difference is a constant.

1, 2, 3, 4 . . . . . . fd = 1

2. Quadratic: Second difference is a constant

Eg 1 4 9 16 25 36

3 5 7 9 11 13

2 2 2 2 2

3. Geometric:

The ratio of one term to its predecessor is a constant : 4 ÷ 2 = 2 8 ÷ 4 = 2 etc

$T_n = 2^n$ 2, 4, 8, 16, 32....

Harder Sequences

Generate the first 5 terms of sequences with the rule:

a) $T_n = n^2 - 2n$ T_n : -1, 0, 3, 8, 15

b) $T_n = 3n^2 + 4n - 5$ 2, 15, 34, 59, 90...

c) $T_n = 2^{n-1}$ $2^0 = 1$ 1, 2, 4, 8, 16

d) $T_n = \dfrac{10}{n}$ 10, 5, $3.\dot{3}$, 2.5, 2 .....

Quadratic sequences

Find the n^{th} term for this quadratic sequence:

4, 15, 32, 55, 84........

n	1	2	3	4	5
	4	15	32	55	84
fd	11	17	23	29	
sd		6	6	6	
$3n^2$	3	12	27	48	75
$2n-1$	1	3	5	7	9

$T_n = 3n^2 + 2n - 1$

Make one for your neighbour.

n	1	2	3	4	5
$3n^2$	3	12	27	48	75
$+2n$	2	4	6	8	10
-1	-1	-1	-1	-1	-1
T_n	4	15	32	55	84

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B1, B3, B5, C2, C4, C6, C8, C10

Proofs

Algebraic method:

Generalising using algebra

By using a counter- example:

Any example that contradicts the theorem disproves it.

Ex: Prove $2n$ is even for all values of n.

When $n = 2.5$

$2n = 5$ which is odd. $\therefore$ ✗ $\therefore$

$\therefore$ It is false.

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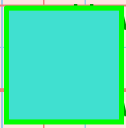
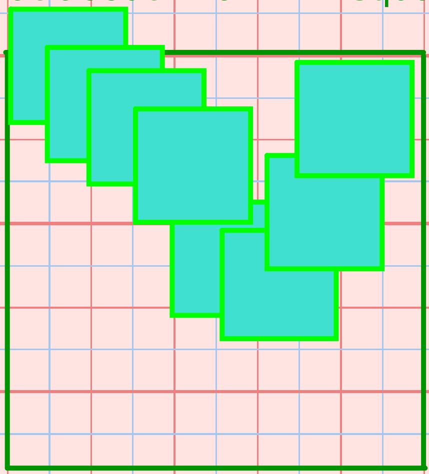


w many 3x3 squares are there in
0x10 square?
w many 3x3 squares are there in
an 11x 11 square?
What about in an nxn square?

Investigate!



In a 10x10 square
there are 64 smaller squares

In an 11x11 square
there are $(11-2)^2 = 9^2 = 81$

In a nxn square
$(n-2)^2$ small squares

We take away 2 because there are colomns at each side where there is no overlapping

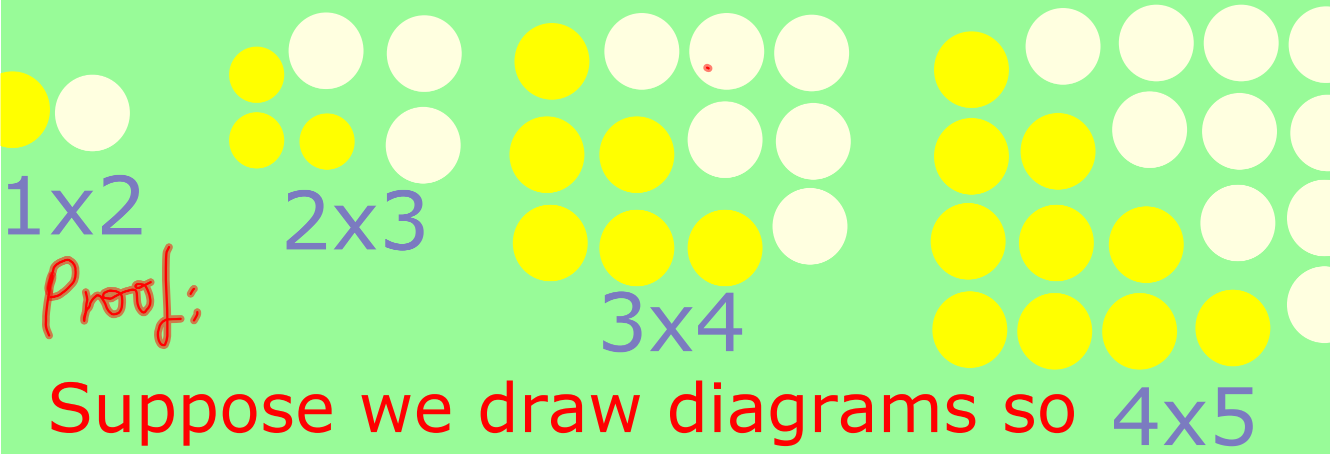
In an 10x10 square there will be 49
4x4 squares. You will get 7 down + 7
across

In a mxm square: $(m-3)^2$ 4x4 square

Tower of Hanoi

see attachment

A proof of the triangular numbers



Proof:

Suppose we draw diagrams so that we double the triangular nos making rectangles as shown.

Each rectangle is n high and (n+1) dots wide.

So in each rectangle there are n(n+1) dots.

Therefore in the triangle there will be 1/2n(n+1) dots. QED

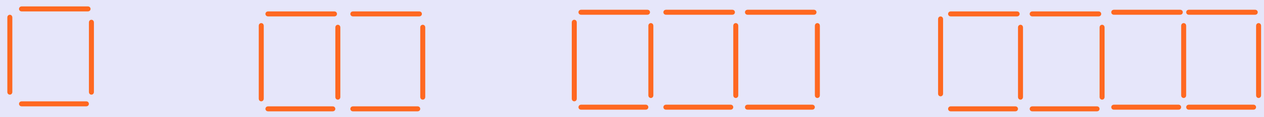
$$(n+2)^2 - n^2$$

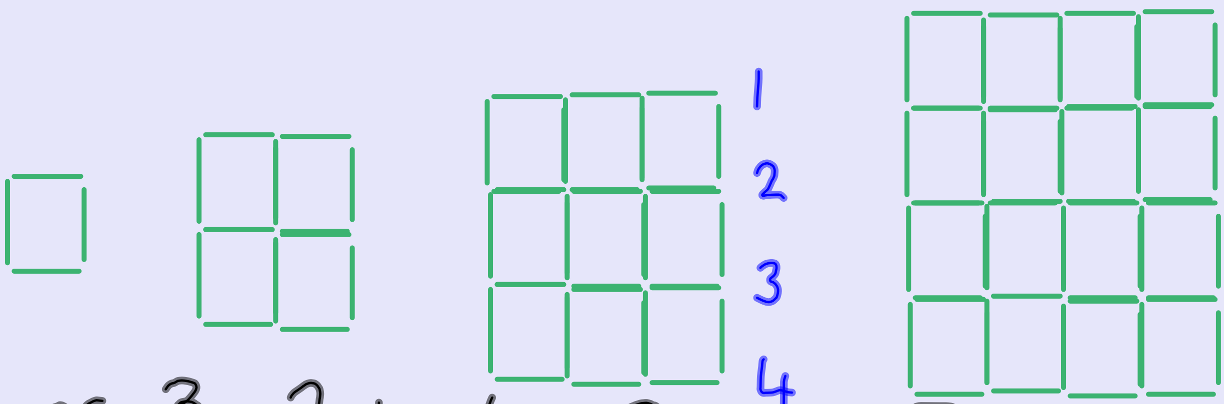
$$= (n+2)(n+2) - n^2$$

$$= n^2 + 4n + 4 - n^2$$

$$= 4n + 4$$ $T_w = 4w + 4$

Finding Rules using structure





row
col
2×1
$+2 \times 1$

$3 \times 2 +$
3×2

4×3
4×3

$5 \times 4 +$
5×4

$6 \times 5 + 6 \times 5$

$(n+1)n + (n+1)n$

$= 2n(n+1)$

$= 2n^2 + 2n$

More algebraic proofs

State whether this is true of false, and give a convincing argument....

The sum of any three consecutive numbers is divisible by 3.

page 372 odd numbers only

Let the first number be n
the second number be $n+1$
the third number be $n+2$

$$n+(n+1)+(n+2)=3n+3$$
$$=3(n+1)$$

Which is a multiple of 3 QED.

b) Let one even no be $2n$
and the other $2m$

$$2n-2m=2(n-m)$$

which is even !

☺ QED

c) $(2m+1)(2x+1)$

$$=4mx+2x+2m+1$$
$$=2(2mx+x+m)+1$$ which is odd QED

For Tues
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Tower of Hanoi (and beyond).ppt